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Stability contracts between supplier and retailer: a new lot sizing model

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Abstract

This work explores the relationship between decision makers in a company and their suppliers, using stability contracts. This relationship can be modeled as a capacitated multi-machine lot sizing problem with minimum order quantity and dynamic time windows, where orders are represented by production levels. Both the amount and the frequency of orders are constrained, the first by upper and lower bounding and the second by dynamic time windows. A mathematical model is provided and an experimental analysis is conducted. Conclusions are given leading to insight for decision makers and contract designers.

Keywords: supply chain, supplier selection, contract, lot-sizing, minimum order quantity, dynamic time window

1 Introduction

With the emergence of globalization and international competition, the outsourcing of parts and services has been steadily increasing over the last decades. Consequently, the relationship between decision makers and suppliers becomes one of the most important issues for supply chain design.

The supplier selection phase is the first step in the relationship between decision makers and suppliers. Efficient collaboration is a clear objective in this relationship [2], and therefore supplier selection must be followed by a good management of the relationship. Bad collaboration can have disastrous consequences: the Bullwhip effect is a good example of what can happen without any information sharing [13] [21]. In order for companies to have fruitful relationships with their suppliers, pertinent contracts must be established [15]. Contracts strengthen and define the relationship between the partners, with the objective of establishing a long-term, cost-effective relationship [22].

The sizes of each company have to be taken into account when establishing a contract. Considering the example of an asymmetric partnership between 2 actors, Blomqvist *et al.* [8] bring up the importance of trust in the relationship. They defined 5 requirements to have trust; their case studies show that every time a given partnership comes to end, one of these requirements had been broken [8].

Since it is an important part of profit coming from the supply chain, industrial actors and researchers have studied contract design in depth. Elmaghraby provides an overview of contract competition in the manufacturing supply chain, studying supplier selection and additionally giving a detailed a review of contract management [15]. The ethic point of view has been recently studied by Eckerd and Hill, focussing on the relationship between buyer and supplier, within a large review [14].

Some contracts take the price discount directly into account. This is a common business strategy since it allows the reinforcement of the partnership, as well as a way to reward special effort. Bassok and Anupindi modeled the price discount in a simple supply chain of one supplier and one retailer with a minimum commitment. With optimal policy definition, they were able to find the best compromise between minimum commitment and price discount [6]. Quantity discount can be balanced by lead time-dependent discount, with powerful simulation tools that allow the study of hypothetical models as close to the real situations as possible [25].

Contracts have to deal with procurement policy. To ensure that a contract leads to a long-term relationship, it must guarantee regular orders and a certain amount of supplied components. In this paper, we consider that a company has to procure a single product from a set of suppliers. In this case, the problem can be seen as a special case of a multi-machine single-item capacitated lot sizing problem with new constraints. The relationship with each supplier is regulated by a contract in which a principle of regularity of orders must be respected. First, the quantity of items in each order must be within a defined interval. Second, the time interval between two orders must be in a given time window. Finally, all the constraint parameters are different for each supplier.

The single item capacitated lot sizing problem (CLSP) has been widely studied in the last five decades. Bitran and Yanasse [7] reviewed the whole algorithmic complexity of this problem. In their classification, they clarified whether many sub-problems are NP-hard or not, establishing in some cases polynomial algorithms. Florian and Klein studied the capacitated version, considering concave cost functions for both production and storage. They proposed a polynomial time algorithm based on the concept of the optimal sub-sequences [16]. Later, Shaw and Wagelmans [24] established a two-step procedure to solve the CLSP with linear production costs, setup costs and general holdings costs functions. They extended their work to the case in which the production cost functions are piecewise linear (not necessarily convex or concave). A complete survey of the single-item lot sizing problem can be found in [10].

Recently the Minimum Order Quantity (MOQ) constraints have been introduced. These constraints deal with the production level that must be at least equal to the MOQ if production is started. Constantino [12], from a polyhedral point of view, considered the production level as a continuous variable. He included the MOQ constraint where the capacity is shared for all the machines. He derived strong inequalities which describe the convex hull of the solutions. Chan and Muckstadt [11] studied a production-inventory system in which the production quantity is constrained by a minimum and a maximum level in each period. They characterized the optimal policy for finite and infinite time horizons, confirmed by an experimental study. The first exact polynomial time algorithm was recently developed by Okhrin and Richter [23]. They solved a special case of the problem in which the unit production cost is constant over the whole horizon, and can therefore be discarded. Furthermore, they assumed that the holding costs are also constant over the T periods. Considering these restrictions, they derived a polynomial time algorithm in $O(T^3)$. Hellion *et al.* [17] developed an optimal $O(T^5)$ polynomial time algorithm to solve the CLSP-MOQ with concave costs functions generalizing Okhrin and Richter's problem.

The MOQ and capacity constraints are not the only way to define stability contracts. The regularity in time is also an option, and can be seen as time window constraints on the supply periods. In the existing literature, time windows have been introduced with several definitions. The delivery time window (also called grace period) was first presented by Lee *et al.* [20]. In their model, each demand i must be delivered during a time window. Later, Akbalik and Penz [4] used a similar definition to compare just in time and time windows policies. The production time windows were introduced by Brahimi *et al.* [9]. In this problem, items cannot be produced before a defined period. Recently, Absi *et al.* [1] studied two production time window problems, considering lost sales or backlogs. They used dynamic programming to solve their problems. Hwang [18] proposed an $O(T^5)$ algorithm for the production time windows and concave production costs. Van den Heuvel [27] showed that the formulations with production time windows are equivalent to other models: lot sizing with manufacturing options, lot sizing with cumulative capacities and lot sizing with inventory bounds. These two time window definitions were studied by Wolsey [28], by proposing valid inequalities and convex hulls. The time windows considered in this paper are different from those defined in the literature: here, the time window that constrains a supply order is dependent on the period where the last order has been sent. These time windows are denoted dynamic time windows in the following.

As discussed above, each company's supplier can be seen as a single machine in the lot sizing problem, leading to a *multi-machine* CLSP. Akbalik and Penz [3] studied a multi-machine CLSP with piece-wise linear productions costs. They derived an exact pseudo-polynomial dynamic programming algorithm which makes their problem NP-hard in the ordinary sense. Bai and Xu [5] studied a single product CLSP in which the retailer may replenish his inventory from several suppliers. Their suppliers present original cost structure characterized by one of two types of order cost structures: incremental quantity discount cost structures and multiple setup cost structures. They derived dominant properties for this problem, and proposed several dynamic programming algorithms. Later, Toktas-Palut and Ulengin [26] tried to coordinate the inventory policies in a decentralized supply chain with multi-supplier. They developed centralized and decentralized models, and then derived contracts allowing to coordinate the supply chain.

In this study, we present a lot sizing problem with a set of new constraints. A literature review showed that the dynamic time window constraints have never been studied before. In this paper stability contracts are defined, with the introduction of a new contract based on dynamic time windows, leading to a new lot sizing problem. This lot sizing problem is analysed, and an experimental study is made; the resolution efficiency is not investigated, however. The paper is organized as follows: Section 2 describes the problem and introduces mathematical formulation. Section 3 describes the testing of the model. The design of experiment is detailed, followed by a cost analysis. This section finishes by studying the strong link between the capacity constraints and the time windows. Finally, concluding remarks and perspectives are given in section 4.

2 Problem description and mathematical formulation

2.1 Description

The single item lot sizing problem consists of satisfying the demands d_t of a product at each period t over T consecutive periods. In our problem, there are S different suppliers (machines in the lot sizing terminology). A demand $d_t \in D$ may be satisfied from the suppliers at period t or from inventory. Backlogs are not allowed. An order cost c_{it} is incurred when a product is ordered from supplier i at period t. The quantity ordered is noted x_{it}. The inventory level at the end of a period t is denoted s_t and the unit storage cost is h_t. A setup cost f_{it} is incurred only if an order is placed to the supplier i at period t. It is assumed that there is no inventory at the beginning of the first period. The problem is to determine the amount x_{it} to be ordered from each supplier, at each period, satisfying the demands and minimizing the total cost.
Each supplier i has a specific contract with the decision maker. For each supplier i, the quantity of products in each order is constrained by a maximum capacity level U_i. The quantity ordered is also constrained by a minimum order quantity L_i. Each subsequent order for supplier i is also constrained by a dynamic time window. There are at least Q_i and at most R_i periods between two consecutive orders. Figure 1 illustrates the dynamic time window for $Q = 2$ and $R = 4$. In the example, an order is placed at period t. Since $Q = 2$, the following order cannot be placed at period $t + 1$ nor $t + 2$. Since $R = 4$, at least one order must be placed in the next five periods. Thereafter, the next order must be placed between $t+3$ $(t+Q+1)$ and $t+5$ $(t+R+1)$ included. Consequently, in each interval of length 3 at most one order must be placed. Furthermore, in each interval of length 5, at least one order must be placed. Note that if $Q = R$ an order must be placed at every $Q + 1$ periods. If $Q = R = 0$, an order must be placed at every period.

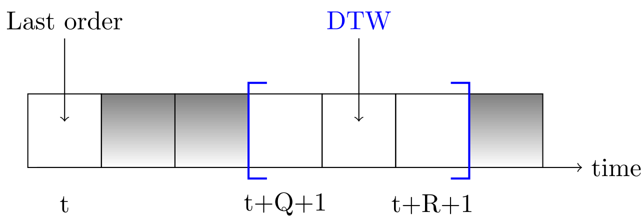


Figure 1: An example of dynamic time window with $Q = 2$ and $R = 4$

The one period sub-problem was proved NP-hard in the ordinary sense by Akbalik and Penz [3] by a reduction from the knapsack problem. Consequently, the general problem is also NP-hard. A simple mixed-integer linear program approach (MILP) is adopted, since the efficiency of the method is not investigated in this paper. The main objective is to understand the impact of the different parameters and the behaviour of the optimal solutions.

2.2 A MILP formulation

A MILP formulation can now be presented. The decision variables are as follows:

- x_{it}: quantity of products ordered from supplier i at period t.
- $y_{it} = \begin{cases} 1 & \text{if an order is placed to the supplier } i \text{ at period } t. \\ 0 & \text{otherwise.} \end{cases}$
- s_t = inventory level at the end of a period t.

Lower and upper bounds on the ordered quantity (MOQ constraints) are integrated into the model, by adding the following constraints:

$$L_i y_{it} \leq x_{it} \leq U_i y_{it}$$

The dynamic time window (DTW) constraints can be written as follows:

$$\sum_{t'=t}^{t+R_i} y_{it'} \geq 1 \qquad \sum_{t'=t}^{t+Q_i} y_{it'} \leq 1$$

Finally, the mathematical formulation is as follows:

$$\text{Min} \sum_{i=1}^{S}\sum_{t=1}^{T} c_{it}x_{it} + \sum_{t=1}^{T} h_t s_t + \sum_{i=1}^{S}\sum_{t=1}^{T} f_{it}y_{it} \tag{1}$$

$$\sum_{i=1}^{S} x_{it} + s_{t-1} - s_t = d_t \qquad \forall t \in \{1, \cdots, T\} \tag{2}$$

$$x_{it} - U_i y_{it} \leq 0 \qquad \forall t \in \{1, \cdots, T\}, \forall i \in \{1, \cdots, S\} \tag{3}$$

$$x_{it} - L_i y_{it} \geq 0 \qquad \forall t \in \{1, \cdots, T\}, \forall i \in \{1, \cdots, S\} \tag{4}$$

$$\sum_{t'=t}^{t+R_i} y_{it'} \geq 1 \qquad \forall t \in \{1, \cdots, T - R_i\}, \forall i \in \{1, \cdots, S\} \tag{5}$$

$$\sum_{t'=t}^{t+Q_i} y_{it'} \leq 1 \qquad \forall t \in \{1, \cdots, T - Q_i\}, \forall i \in \{1, \cdots, S\} \tag{6}$$

$$x_{it} \in \mathbb{R} \qquad \forall t \in \{1, \cdots, T\}, \forall i \in \{1, \cdots, S\} \tag{7}$$

$$y_{it} \in \{0, 1\} \qquad \forall t \in \{1, \cdots, T\}, \forall i \in \{1, \cdots, S\} \tag{8}$$

$$s_t \in \mathbb{R} \qquad \forall t \in \{1, \cdots, T\} \tag{9}$$

The objective function (1) is to minimize the production, holding and setup costs, respectively. Constraint (2) is the flow constraint. Constraints (3) and (4) define the maximum and minimum order quantities. The dynamic time windows are given by (5) and (6). Constraints (7), (8) and (9) define the validity domain of each variable.

3 Computational experiment and analysis

All the following experiments have been performed on a 3 GHz Intel Core 2 Duo Machine with 4GB memory using IBM ILOG CPLEX 12.2 on Windows 7.

3.1 Design of experiment

To study this problem, we use the *design of experiment* methodology [19] to extract the key factors in the process. The design of experiment can also draw out the interactions of two or more factors. To plan a two-level full factorial design of experiment, all the factor have to be determined. All the costs have to be considered: storage cost, and for each supplier purchasing cost and setup cost. The contract parameters are L_i, U_i, Q_i and R_i for each supplier i. Assuming that the demands are generated by a normal law, the mean μ and the variance σ are also factors of the model.

In this experiment two suppliers are considered, namely A and B. The contract defined by L, U, Q and R is between the supplier A and the retailer. The contract with the supplier B does not constrain the order quantities nor the order frequency, *i.e.* $Q = 0$, $R = T$, $L = 0$ and $U = \infty$, thus B's purchasing cost is greater. Since the demands must be satisfied, and the total costs are minimized, supplier A's purchasing costs can be discarded. Both suppliers share the same setup costs. The two-level full factorial design of experiment parameter values are summarized in Table 1.

Since 9 parameters are studied, 2^9 parameter combinations have to be performed. For each parameter combination (referred to as sets herein), 25 instances are generated and solved. Solving a single instance takes less than 2 seconds, but considering the number of instances is 12800, the whole experiment takes about 5 hours to compute.

	parameter	level 1	level 2
R	maximum number of periods between two orders from supplier A	3	T
Q	minimum number of periods between two orders from supplier A	0	3
U	maximum amount in one order from supplier A	50	$\sum_{\forall t} d_t$
L	*minimum order quantity* for the supplier A	0	30
σ	demand variance	2	10
μ	demand mean	5	20
f	setup production cost for both suppliers	2	10
c	supplier B unit production cost	0.1	4
h	storage cost	0.1	1

Table 1: *Design of experiment* parameters and their values

The main effects of each parameter are summarized in Figure 2. The largest effect comes from the storage cost, its maximum value strongly affects the value of the solution. Generally speaking, a low storage cost allows the decision maker to deal with contract constraints, by storing more products. An additional experiment based on the inventory level is performed in section 3.3 focussing on storage cost.

It is interesting to note that the setup cost (*i.e.*, paying 10.0 for each order instead of 2.0) does not appear to have a strong effect on the solution. In some way, the Q and L constraints play the role of setup cost. This *design of experiment* proves that the setup cost can be discarded from this problem without changing the behavior of the model. Consequently the next experiments in Section 3.2 and 3.3 will not include the setup cost.

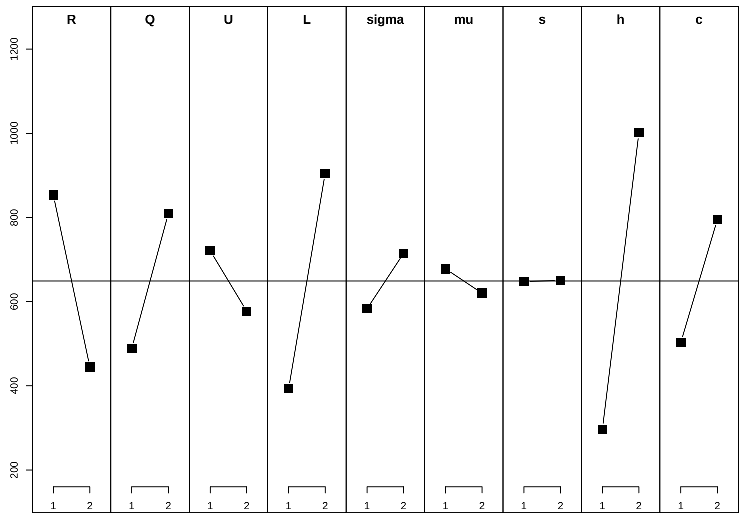


Figure 2: Main effects

We denote as *min* and *max* the minimum and maximum amount of products which can be ordered, respectively. Figure 3 displays the effect of each parameter R, Q, U and L on *min* and *max*. For example, taking the maximum value of Q ($Q_{1\to2}$) decreases *max* ($\Searrow$), and has no effect on *min* (-). Each parameter has different effects on *min* and *max*, and these effects are summarized in Figure 3. For instance, if the decision maker wants to decrease *max* of the contract, he can choose either to increase Q ($Q_{1\to2}$) or to decrease U ($U_{2\to1}$). All other choices do not decrease *max*.

parameter	*max*	*min*
$Q_{1\to2}$	$\Searrow$	-
$R_{1\to2}$	-	$\Searrow$
$L_{1\to2}$	-	$\Nearrow$
$U_{1\to2}$	$\Nearrow$	-

Figure 3: R, Q, U, L parameter effects on *min* and *max*

3.2 Impact of the demand variance on the supply policy

In this experiment, three suppliers are considered with different contracts. We study how the demand variance affects the retailer's supply. More precisely, we compare the amount of product ordered from the 3 suppliers, A, B and C, when the demand variance is increasing.

A contract A dominates a contract B ($A \geq_d B$) if for all parameters, B is more restrictive than A. Formally,

$$A \geq_d B \equiv \begin{cases} L_A \leq L_B \\ U_A \geq U_B \\ Q_A \leq Q_B \\ R_A \geq R_B \end{cases}$$

In our case, $A \geq_d B \geq_d C$. In addition, being more restrictive naturally implies lower cost, consequently $c_A \leq c_B \leq c_C$. The contracts are summarized in Table 2.

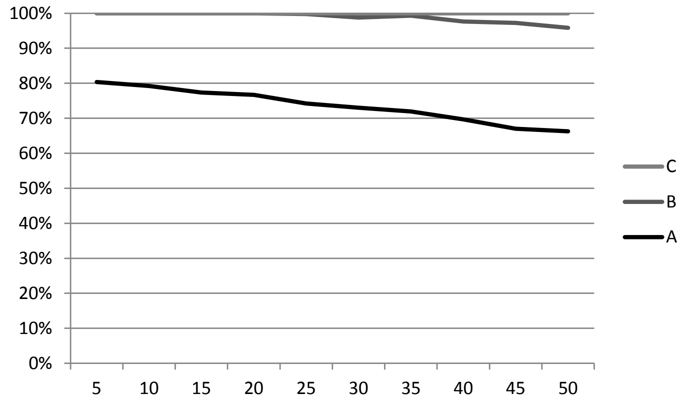
num	A	B	C
Q	0	2	0
R	0	5	T
L	30	10	0
U	40	100	D
c	1.0	1.1	1.6

Table 2: Parameters of the contracts

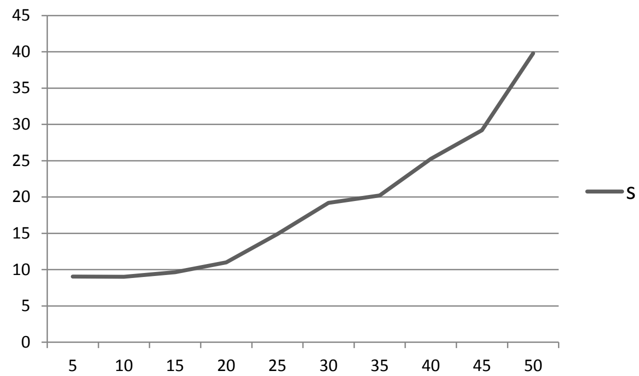
Demands are generated from a normal distribution of mean $\mu = 50$ and variance starting from 5, up to 50 with a step of 5. The number of periods T is 50. Storage costs are fixed to 0.1. For each value of variance, a set of 20 instances is randomly generated. The mean of the total demand is $50 \times 50 = 2500$ products. Results are summarized in Figure 4a and 4b.

In Figure 4a, the stacked proportion of products ordered from each supplier is shown. Each value is the mean of a set of 20 instances. The minimum and maximum proportion of products which can be ordered from supplier A on the whole horizon is 1500 and 2000, respectively, *i.e.*, 60% and 80% of the total demand. Starting at 80%, the proportion of products ordered from supplier A decreases to reach almost 65% of the total demand, which is almost the minimum amount of product which must be ordered (60%). At the same time, the proportion of B and C varies, increasing from 20% to 30% for B, from 0% to 5% for C.

A way to deal with high variance is the increase of the storage. This is shown in Figure 4b, which displays the inventory level when the variance increases. Starting lower than 10, this average increases to reach 40. Another solution is to order from another supplier, with higher production cost. The Figure 4a shows that as demand variance increases, other suppliers are used, despite their higher production cost. This implies a slight cost increase, starting from 2500 (for the lowest variance), up to 2770 (for the greatest variance).



(a) Suppliers allocation (in percentage) as variance increases



(b) Average inventory level in one period (in number of pieces) as variance increases

Figure 4: Impact of the demand variance on the supply policy

3.3 Impact of tightening bounds

The next two experiments aim to study the effects of the contracts on the storage cost. Additional costs like transportation costs, production costs, setup costs are not considered. This allows us to make profound observations of the impacts of these contracts on the storage cost. We only consider a single supplier. The original objective function is as follows:

$$\text{Min} \sum_{i=1}^{S} \sum_{t=1}^{T} c_{it} x_{it} + \sum_{t=1}^{T} h_t s_t + \sum_{i=1}^{S} \sum_{t=1}^{T} f_{it} y_{it} \tag{10}$$

Assuming that the unit production costs are constant, and that the backlogs are not allowed, the term $c_t x_t$ can be omitted from the objective function (10), because it is constant regardless of the ordering. Furthermore, instead of the setup cost, we specify the minimum order quantity, which plays the role of a minor setup cost, as shown in Section 3.1. If we additionally assume that the holding costs are also constant, then the objective function is reduced to a single term that minimizes the cumulative inventories (11). These assumptions have been recently used in [23]. The objective function 11 will be studied in the rest of the paper:

$$\text{Min} \sum_{t=1}^{T} s_t \tag{11}$$

For the supplier, the quantity of products that can be ordered at each period is constrained by the maximum constant capacity level U. The quantity ordered is also constrained by a constant minimum order quantity L.

The contracts can be constrained in two different ways: by tightening the capacity and MOQ constraints, or by tightening the time windows. In the following two experiments, one part of the contract is fixed while the other one varies. Only feasible instances should be solved, and thus a lower and upper bound on the demands are necessary. Uniform distributions are the simplest way to generate the demands.

3.3.1 Cost impact of tightening one contract part

First of all, we will study the quantity part of a contract, *i.e.*, keeping the time windows constant, increasing L and decreasing U. Considering one fixed couple $\{Q, R\}$, and picking 10 different couples $\{L, U\}$ (Table 3c), with 3 uniform distributions (Table 3a), 100 randomly instances are generated for each set; 3000 instances are generated this way. All these instances are solved for 2 different couples $\{Q, R\}$ (Table 3b).

a	b
75	125
50	150
25	175

(a) Uniform distribution $\mathcal{U}(a, b)$

Q	R
0	10
3	7

(b) $\{Q, R\}$ fixed

L	U
100	1000
150	950
200	900
250	850
300	800
350	750
400	700
450	650
500	600
550	550

(c) $\{L, U\}$ Tightened

L	U
100	1000
400	700

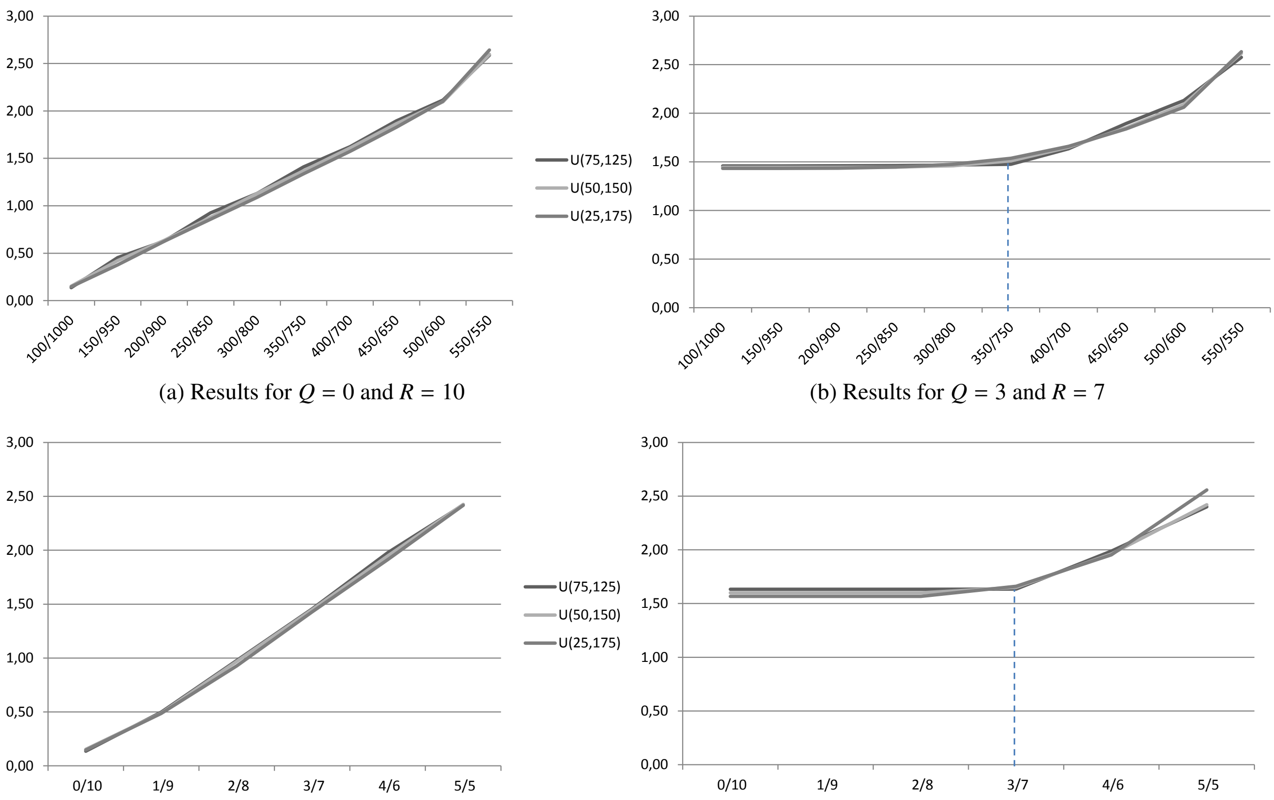
(d) $\{L, U\}$ fixed

Q	R
0	10
1	9
2	8
3	7
4	6
5	5

(e) $\{Q, R\}$ Tightened

Table 3: Experiment parameters

Figures 5a and 5b show that increasing the uniform distribution variance does not affect the solution. Indeed the three variance-dependant curves are close. Starting with a unconstrained contract: $\{Q, R\} = \{0, 10\}$, tightening the $\{L, U\}$ part makes the graph linear (Figure 5a). The differences appear when the fixed part of the contract is already tightened *i.e.* $\{Q, R\} = \{3, 7\}$: see Figure 5b. In this case, the graph is almost constant from $\{L, U\}$ = $\{100, 1000\}$ to $\{400, 700\}$, as shown Figure 5b. The costs are stable at 1.5 values per



(a) Results for $Q = 0$ and $R = 10$

(b) Results for $Q = 3$ and $R = 7$

(c) Results for $L = 100$ and $U = 1000$

(d) Results for $L = 400$ and $U = 700$

Figure 5: Cost impact of one contract part tightening

product (dotted lines).

Inversely to the experiment above, fixed amount constraints (L and U) are considered and time windows (Q and R) are now tightened. Considering one fixed couple $\{L, U\}$, picking 6 different couples $\{Q, R\}$ (Table 3e), with 3 uniform distributions (Figure 3a), 100 randomly instances are generated for each set, thus 1800 instances are generated. All these instances are solved for 2 different couples $\{L, U\}$ (Table 3d).

As in the previous case, Figures 5c and 5d show that the different variances do not affect the solution. As previously, starting with a unconstrained contract: $\{L, U\} = \{100, 1000\}$ makes the graph linear (Figure 5c). Furthermore, if $\{L, U\}$ is tightened *i.e.* $\{L, U\} = \{400, 700\}$, the cost is also constant until a $\{Q, R\}$ is $\{3, 7\}$ (Figure 5d).

The two experiments performed above, *i.e.*, tightening $\{L, U\}$ contract part fixing $\{Q, R\}$ and tightening $\{Q, R\}$ fixing $\{L, U\}$ produce similar graphs. If a part of a contract is already tightened, tightening the second part costs nothing for the retailer, to a certain extent: the product cost stays at 1.5. In this situation, if the retailer negotiates a cost variation of 1.5, it will be enough to cover all the storage costs. Mathematically, if a part of a contract is tightened and then fixed, at least one solution among the optimal solutions directly implies a tightened second part of the contract.

3.3.2 Cost impact of tightening both contract parts

The experiment above has showed that the use of a different uniform law does not affect the solution. In this experiment, only one uniform law is considered, the others are discarded.

The experiment above has highlighted the relationship between the two parts of a contract. The next experiment aims to go further, defining zones where a contract part can be tightened without almost any additional cost. The previous experiment unilaterally tightened a part of a contract, *i.e.* increasing L (or Q) and decreasing U (or R) at the same time. In the following all the combinations respecting $L \leq U$ and $Q \leq R$ are tested.

As previously, one of the contract parts is fixed while the other part varies. Three contracts are fixed for each part, leading to 6 fixed contracts *i.e.* 6 different sets. Each set is presented separately. For each set all the contracts are tested, and the average storage cost for one single product is compared. The percentages are calculated from the least tightened contract of each set (the bottom left corner of each figure). This way, for each cost variation (even if it is zero), a set of equivalent contracts (at same cost for the retailer) can be found.

300	49,67%	49,96%	50,88%	53,35%	104%	
350	3,54%	3,86%	4,68%	7,99%	24,45%	85,49%
400	0,41%	0,75%	1,52%	4,85%	19,86%	48,04%
450	0,02%	0,38%	1,12%	4,44%	19,24%	44,37%
500	0%	0,36%	1,1%	4,42%	19,2%	44,07%
550	0%	0,36%	1,1%	4,42%	19,2%	43,99%
600	0%	0,36%	1,1%	4,42%	19,2%	43,99%
650	0%	0,36%	1,1%	4,42%	19,2%	43,99%
700	0%	0,36%	1,1%	4,42%	19,2%	43,99%
750	0%	0,36%	1,1%	4,42%	19,2%	43,99%
800	0%	0,36%	1,1%	4,42%	19,2%	43,99%
850	0%	0,36%	1,1%	4,42%	19,2%	43,99%
900	0%	0,36%	1,1%	4,42%	19,2%	43,99%
950	0%	0,36%	1,1%	4,42%	19,2%	43,99%
1000	0%	0,36%	1,1%	4,42%	19,2%	43,99%
U,L	100	150	200	250	300	350

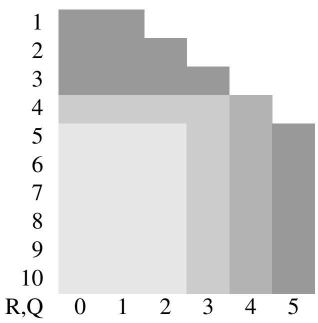
Figure 6: {Q,R} = {2,8}

The 6 fixed contracts are $\{L, U\} = \{300, 800\}$, $\{400, 700\}$, $\{500, 600\}$ (Figures 7, 8a, 8b) and $\{Q, R\} = \{2, 8\}$, $\{3, 7\}$, $\{4, 6\}$ (Figures 6, 9a, 9c). For $\{L, U\}$ fixed contract, Q varies between 0 and 5, whilst R starts at Q, up to 10. For $\{Q, R\}$ fixed contract, L varies between 100 and 500, whilst R starts at L, up to 1000. The demands are generated with a single uniform law $\mathcal{U}(50, 150)$ which implies that the average demand for a period is 100. 50 instances are randomly generated, and these instances are solved for every contract. In the figures, the holes represent unfeasible contracts, either because in this contract $Q > R$, $L > U$ or because the demands cannot be satisfied: $U \times \lceil \frac{T}{Q+1} \rceil < D$.

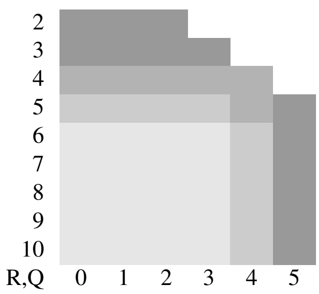
Solving a single instance takes less than 2 seconds, however the number of instances is about 30000, therefore the whole experiment takes about 10 hours to compute. The

R,Q	0	1	2	3	4
2	55,76%	55,76%	56,78%		
3	0,07%	0,07%	1,39%	32,55%	
4	0%	0%	1,31%	28,8%	76,53%
5	0%	0%	1,31%	28,73%	73,16%
6	0%	0%	1,31%	28,73%	73,07%
7	0%	0%	1,31%	28,73%	73,06%
8	0%	0%	1,31%	28,73%	73,06%
9	0%	0%	1,31%	28,73%	73,06%
10	0%	0%	1,31%	28,73%	73,06%

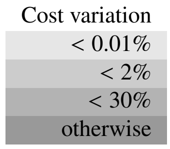
Figure 7: {L,U} = {300,800}



(a) {L,U} = {400,700}



(b) {L,U} = {500,600}



(c) Used colours

Figure 8: Tightening contracts

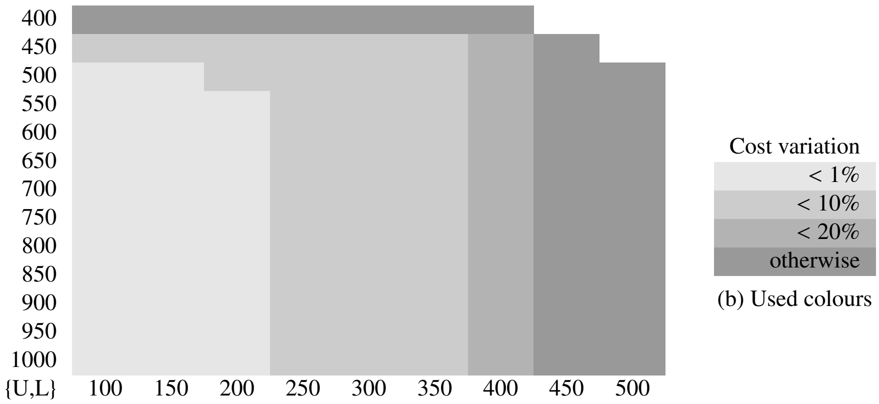
percentages are only displayed for Figures 6 and 7. The percentages in the other figures are not shown for readability, and are instead replaced by shades of grey, which are detailed in Figures 8c and 9b.

The experiments show the presence of certain zones of cost variation. For a given cost variation, say 1% in the Figure 6, all the contracts satisfying $100 \leq L \leq 150$ and $400 \leq U \leq 1000$ have the same price to the retailer. Therefore fixing $\{Q, R\} = \{2, 8\}$ (the assumption of Figure 6) implies that the most constrained contract for this cost variation is $\{L, U\} = \{150, 400\}$. In the following we denote the most constrained contract with a given cost variation as the "best" contract.

Consider a cost variation of 0.01%, in Figure 7; all the contracts satisfying $0 \leq L \leq 1$ and $4 \leq R \leq 10$ have the same price to the retailer. Therefore fixing $\{L, U\} = \{300, 800\}$ (the assumption of Figure 7) implies that the best contract for the same cost is $\{Q, R\} = \{1, 4\}$.

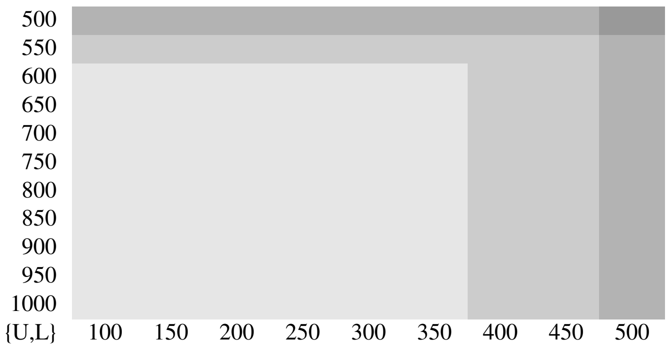
Furthermore if a cost variation of 0.01% is considered, in Figures 6, 9a and 9c, the best contracts with this assumption are not the same. Considering $\{Q, R\} = \{2, 8\}$ (Figure 6), as seen above the best contract is $\{L, U\} = \{150, 400\}$. Considering $\{Q, R\} = \{3, 7\}$ (Figure 9a) the best contract is $\{L, U\} = \{200, 550\}$, whilst for $\{Q, R\} = \{4, 6\}$ (Figure 9c) the best contract is $\{L, U\} = \{350, 600\}$.

Consider a cost variation of 2% in Figures 7, 8a and 8b. As seen before, considering $\{L, U\} = \{300, 800\}$ (Figure 7) the best contract is $\{Q, R\} = \{2, 3\}$, while considering $\{L, U\} = \{400, 700\}$ (Figure 8a) and $\{L, U\} = \{500, 600\}$ (Figure 8b), the best contracts are



(a) {Q,R} = {3,7}

(b) Used colours



(c) {Q,R} = {4,6}

Figure 9: Tightening contracts

$\{Q, R\} = \{3, 4\}$ and $\{Q, R\} = \{4, 6\}$, respectively.

In conclusion, for any cost variation (even 0%), as one part of a given contract is constrained, the other part corresponding to the *best* combination (as defined above) is naturally constrained.

More formally, for any cost variation, for two fixed contract parts $\{Q_1, R_1\}$ and $\{Q_2, R_2\}$, their best other contract parts are noted $\{L_1^*, U_1^*\}$ and $\{L_2^*, U_2^*\}$.
If $Q_2 \geq Q_1$ and $R_2 \leq R_1$ (*i.e.*, contract 2 is more constrained than contract 1), then $U_2^* \leq U_1^*$ and $L_2^* \geq L_1^*$. The same argument can also be used for any fixed contract part $\{L, U\}$.

4 Conclusion

This paper has focused on a particular set of contracts between suppliers and retailers. Our interest is to strengthen and regularize the relationships between these actors. The contracts studied are divided in two parts: the constraints on order amount and the constraints on order frequency. Constraints on the order amount have been deeply studied and are known in literature as production capacity and minimum order quantity (MOQ). Constraints on the order frequency are introduced, and formally defined. We have called these constraints *dynamic time windows*, in contrast with both production and delivery time windows, already studied in literature. According to our knowledge, these dynamic time windows and these contracts have never been considered before.

First, a two-level full factorial *design of experiment* helps to highlight the important parameters of the model. It shows the negligible importance of the setup cost in the problem. The next experiment has studied the supplier selection by increasing the demand variance. Finally, the last experiments have studied the effects that a contract part can have on the other part, and help to understand the strong link between these two parts. Contracts can be exploited from several perspectives, which are detailed below.

The retailer establishes a contract to stabilize the relationship with his supplier. This way, the retailer can convince the supplier to apply a price discount in return for his customer loyalty. This price discount for one product has to be at least equal to the cost variation of this product, in respect with the contract constraints.
The supplier would like to retain its customer, and thus offers the retailer a price discount in return for the respect of the contract, guaranteeing him an upper bound for the additional storage costs.
The supplier and the retailer establish a contract to guarantee each other sustainable relationship. A contract could be established to minimize the total costs of the supply chain and possibly result in a price discount for the final customer.

The efficiency of the method is not investigated in this paper. Without considering time window, many variants of the problem have been shown to be polynomial [16], [23], [17]. Further work will focus on finding out the theoretical complexity of the studied problem, as well as determining the different assumptions which keep the problem polynomial.

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