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Stability Contract in the Forest Products Supply Chain: Case Study for a Quebec Region

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Abstract

In this paper an industrial case including a papermill and its three suppliers (sawmills) is studied. Sawmills produce lumber and chips from wood, and the paper mill needs these chips to make paper. These sawmills assign a lower priority to the chips market even though the paper mill is their main customer. The focus of the research is therefore on securing the paper mill supply by creating beneficial contracts for both stakeholders. Industrial constraints are taken into account, leading to separate contract designs. Contracts are then tested on various instances and compared to a centralized model that optimizes the total profit of the supply chain. Results show that the decentralized profit with separate contracts is 99.3% the centralized profit, for a fixed demand variance. Difference between centralized and decentralized profit slightly increases with the variance, to reach 3% for a variance of 50%.

1 Introduction

Members of the Canadian forest industry agree that the wood market is currently experiencing its worst crisis for a long time. The US is the main final customer for all Canadian wood products. Thus the subprime crisis in 2008 had a disastrous impact on North American investments. In particular, the US lumber demand fell in the following years, causing a lumber price decrease on market from 450$ (CAD) (near the years 2000) to 298$ (CAD) (2009) [9]. The crisis is not the only reason explaining this price decrease. The recent competition of emergent countries such as Chile, Brazil and China is also highlighted.

In this work, we study an industrial case concerning forest product companies located in the Côte-Nord region in Quebec, Canada. Our interest is focused on the last paper mill of this region, and its three main suppliers, which are sawmills. Sawmills operations and the paper making process are linked together but typically managed independently, leading to a profit waste. Furthermore, the paper mill is the only purchaser for all the wood chip produced by nearby sawmills. The shutting down of this paper mill would have unpredictable consequences.

In order to ensure the collaboration between the main stakeholders, a working group gathering all the forest companies has been created [14] to rethink about their business model. Every week the group meets to discuss about how to satisfy the paper mill demand. Each sawmill must share the informations about the volume of chips to send to the paper mill. These informations include the chips freshness and density, which are related to the tree species. As the sawmills have only one customer, they have to adapt their production planning until the global paper mill needs are satisfied [14].

Based on this context, we propose the use of beneficial contracts for the stakeholders in order to secure the paper mill supplies. In this way, it becomes possible to deliver the volume of chips needed while better coordinating network operations.

The relationship between decision makers and suppliers becomes one of the most important issues of the supply chain. To be profitable, supply chain activities need to be better coordinated, necessitating stronger interactions between stakeholders [5]. However, examples of poor collaborations that have disastrous consequences are given by Thomas and Griffin [24].

The so-called Bullwhip effect is a good example of what can happen without any information sharing. Dejonckheere *et al.* [8] studied it from a mathematical and statistical point of view. Lee *et al.* [19] pointed out the Bullwhip effect in the MIT beer game. De souza *et al.* [7] made a large experiment to assess the factors leading to unsuccessful collaborations, highlighting the importance of information sharing.

Collaboration between stakeholders in supply chain is besides a huge subject of interest. Huang *et al.* [18] presented a review in which they conclude that the number of papers about collaboration exploded between 1996 and 2003.

Camarinha-matos *et al.* [4] proposed different classes of collaborative networks reflecting industry's reality. Sahin and Robinson [22] suggested a review including many industrial references. Many authors have also pointed out the fact that collaborations must be guided in order to be profitable for each supply chain member [20]. For example, Prahinsky and Benton [21] showed that if automotive firms demonstrate increased willingness to share information, the supplier's commitment to the relationship also increases. The decision maker should have all the stakeholders's information to better optimize a supply chain. However this is usually operationally unrealistic [16]. Consequently, when knowledge is combined, determining the key informations that have to be shared as well as the profit that may occur have been even more studied during the past years. Dyer and Chu [11] [12] studied information sharing in the car industry, and conclude that firms signal their own trustworthiness through a willingness to share information. Datta and Christopher [6] showed the importance of information sharing between supply chain members to better face uncertainties. Some papers studied the consequences of forecasting error (Zhao and Xie [26]) and information sharing (Yu *et al.* [25]) in real case studies.

Since it is an important step in collaboration implementation, practitioners and academics heavily studied the contract design. Elmaghraby [15] provided an overview of the contract competition in the manufacturing supply chain while Cachon [3] described different types of contract as coordination mechanisms for the supply chain. More recently, trust has been studied as an important part of the supplier-retailer relationship. Eckerd and Hill [13] modeled the relation between buyers and suppliers, from an ethical point of view.

Blomqvist *et al.* [2] found from several case studies that every time the partnership comes to end, a trust rule must have been broken. Trust has also been defined and deeply studied by Doney and Cannon [10]. Researcher as Selnes [23] demonstrated that enhanced communication contributes significantly to customers satisfaction. The size of the stakeholders may also have an impact on collaboration creation and management, leading to specific leadership and ownership models [1].

The context studied in this research concerns the interaction between three sawmills and one paper mill. The paper mill raw material is the chips supplied by the sawmills. In particular, when producing lumber from wood, sawmills generate at the same time chips that can be combined with chemicals to produce pulp and then paper. The paper mill requires a large amount of the wood chips produced by the sawmills, but the latters usually focus on their core business. As a result, sawmills make planning decisions in order to ensure lumber quality rather than chips quality. Chips delivered are therefore variable in terms of volume and quality, leading to higher paper production costs. The purpose of this paper is therefore to secure the paper mill's supply by creating beneficial contracts for both stakeholders.

The paper is organized as follows: Section 2 describes the problem and introduces mathematical notations. Section 3 exposes the formulations used in the paper, for both centralized and decentralized cases. Section 4 includes the centralized model and a cost analysis. The decentralized model and the contract design are studied in Section 5. Contracts are also validated by several experiments on many multi-periods problems with normally distributed demands. Managerial implications and conclusions are given in Section 6.

2 Problem description

The case studied includes two kinds of stakeholders, a paper mill and three sawmills (see Figure 1). All the prices, costs and variables in this paper are based on this industrial context and expressed in m^3.

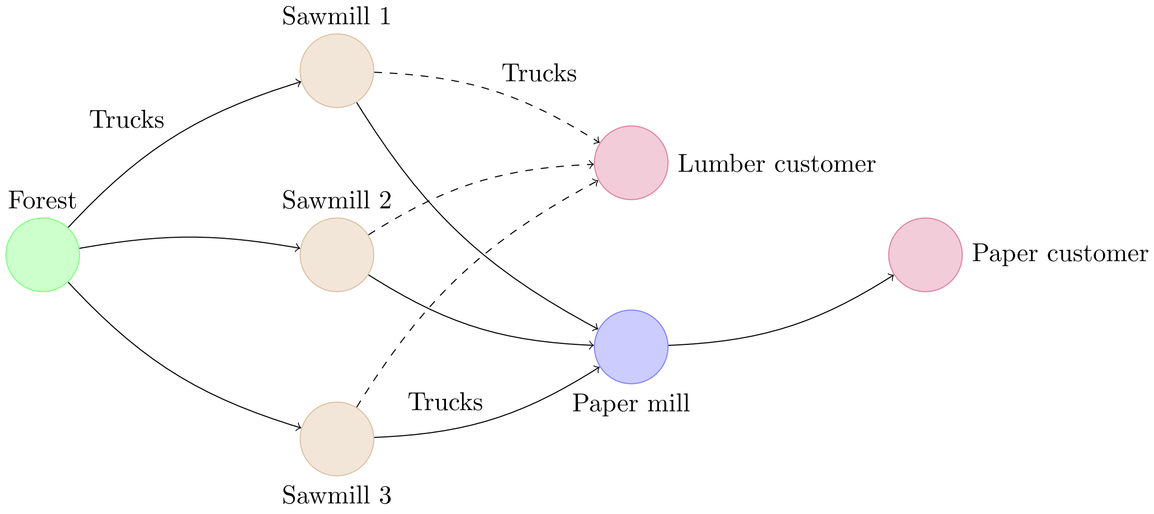


Figure 1: Case study

Sawmill: At each period t, for the sawmill i, wood are delivered by trucks from the forest, at price WP_i. Then the sawmill produces from wood both lumber and chips. The costs for processing wood include harvesting, transportation and sawmilling costs. At each period t, for the sawmill i, lumber are sold to a specific customer at price PB_i, for a maximum demand of $d_{i,t}$, whereas chips are sold to the paper mill at price PC_i. The volume of lumber produced by the sawmill i is constrained by a maximum capacity K_i. The chips produced by the sawmill i has a given quality Q_i. This quality is important for the paper mill and takes value in the real unit interval $[0, 1]$. At the end of each period t, both lumber and chips can be stored at the sawmill i, involving a cost HO_i and HS_i, respectively.

Paper mill: Chips are delivered by trucks from the sawmill to the paper mill. However, these transportation costs are included in the chips price PC_i. At each period t, the paper mill uses these chips to produce paper at a cost of TC, selling it to a specific customer at price PP, for a maximum demand of dp_t. At the end of each period t, both paper and chips can be stored at the paper mill. The paper mill cost reflects chips purchasing and storage, as well as paper production and storage costs (PC_i, HC_i, TC and HP, respectively).

Sawing capacities: Sawmill capacities are given and known. In particular, the capacity of S_1 and S_3 are the same, and they are both twice the capacity of S_2. As a result, they can be substituted in the model as K, $0.5K$ and K, as showed in Table 3.

Chips quality As said before, the chips produced by the sawmills have a given quality, which is associated with the sawmill. This quality takes value in the real unit interval $[0, 1]$ and depends of the humidity, density, and the tree species. The values exposed here have been given by the paper mill.

Constraint MCP: The sawmill process will conduct to both a certain volume of lumber as main product and a minimum amount of chips as co-product. This minimum is denoted as Minimum Chips Proportion (MCP) in the rest of this paper. We denote as **existing chips** the chips that have been produced by the MCP constraint. The chips that have been produced beyond the MCP constraint are denoted as **additional chips**.

Constraint MCQ: To be effective, the paper production requires a minimum chips quality. More precisely, the average quality of chips must be at least a given value. This value is denoted as Minimum Chips Quality (MCQ) in the rest of this paper.

Variables definition: For convenience, the same variables are used in all the models described in this paper. For each period t :

- Wood arriving at the sawmill is noted $W_{i,t}$. Two products are then produced, lumber and chips.
- At a sawmill i, produced, stored and sold lumber are noted $ZZ_{i,t}$, $IO_{i,t}$ and $Z_{i,t}$, respectively.
- Chips are noted $XP_{i,t}$, afterwards they can be stored ($IS_{i,t}$), and then delivered to the paper mill ($X_{i,t}$). Chips can also be stored at the paper mill ($IC_{i,t}$), and then used in paper production ($XT_{i,t}$).
- At the paper mill, produced, stored and sold paper is noted $YY_{i,t}$, $IP_{i,t}$ and $Y_{i,t}$, respectively.

The general process is summarized in Figure 2.

Sawmills, paper mill and customer demands are defined in Table 1. Considered costs are presented in Table 2. Note that in the models, all costs are constant. Industrial constraints and special notations are listed in Table 3. Variables are displayed in Table 4.

Table 1: Notation for the mathematical models

P	paper mill
S_1, S_2, S_3	Sawmills
$d_{i,t}$	demand for lumber of the sawmill i at the period t
dp_t	demand for paper of the paper mill at the period t

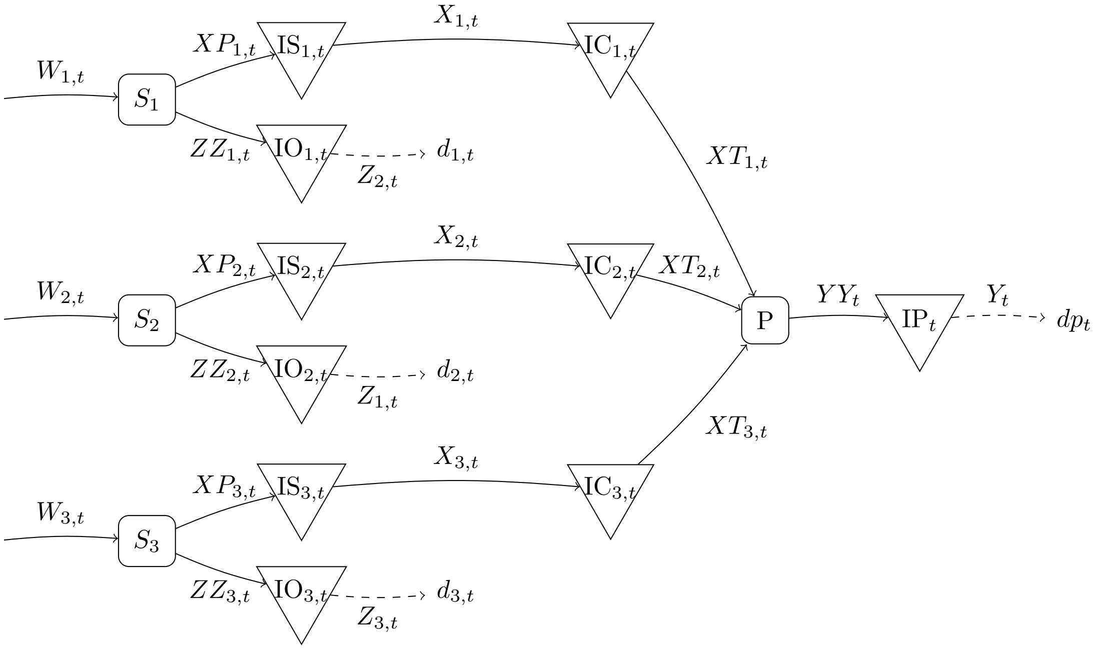


Figure 2: Supply chain modelling

Table 2: Costs

WP_i	Wood production cost per m^3 of S_i	$105\$/m^3$
PC_i	Chip price per m^3 purchased by P at S_i	$56\$/m^3$
PB_i	Lumber price per m^3 sold to satisfy $d_{i,t}$ at S_i, at period t	$142\$/m^3$
PP	Paper price per m^3 sold to satisfy dp_t at P, at period t	$250\$/m^3$
HO_i	Lumber storage cost per m^3 at S_i	$0.38\$/m^3$
HC	Chip storage cost per m^3 at P	$0.13\$/m^3$
HS_i	Chip storage cost per m^3 at S_i	$0.13\$/m^3$
HP	Paper storage cost per m^3 at P	$0.58\$/m^3$
TC	Paper production cost per m^3 by P, using chips	$156\$/m^3$

Table 3: Industrial constraints

Q_1	Chips quality produced by the sawmill S_i	0.6
Q_2	Chips quality produced by the sawmill S_i	0.75
Q_3	Chips quality produced by the sawmill S_i	0.98
MCQ	Chips quality required to transform chips into paper	0.82
MCP	Minimum chips proportion produced by sawmill.	10%
K_1	Sawing Capacity of the sawmill S_1	K
K_2	Sawing Capacity of the sawmill S_2	0.5K
K_3	Sawing Capacity of the sawmill S_3	K

Table 4: Variables

$W_{i,t}$	Amount of wood produced by S_i, at period t
$XP_{i,t}$	Amount of chips produced by S_i, at period t
$X_{i,t}$	Amount of chips provided by S_i to P, at period t
$XT_{i,t}$	Amount of chips uses to make paper by P, originally provided by S_i, at period t
$Z_{i,t}$	Amount of lumber sold by S_i, at period t, in response to demand $d_{i,t}$
$ZZ_{i,t}$	Amount of lumber produced by S_i, at period t
Y_t	Amount of paper sold by P, at period t, in response to demand dp_t
YY_t	Amount of paper produced by P, at period t
$IO_{i,t}$	Amount of lumber stored by S_i, at the end of the period t
$IS_{i,t}$	Amount of chips stored by S_i, at the end of the period t
$IC_{i,t}$	Amount of chips stored by P, originally provided by S_i, at the end of the period t
IP_t	Amount of paper stored at P, at the end of the period t
$L_{i,t}$	Amount of chips lost for the sawmill i and the paper mill, at period t
Lp_t	Amount of chips lost for the paper mill, at period t

3 Mathematical formulations

In this section, the formulations for both the centralized and the decentralized models are presented. The decentralized formulation is used to test contracts, while the centralized formulation is an upper bound for the profit of the whole supply chain. The latter plays the role of a reference for assessing contracts determined via the decentralized model.

- The centralized model optimizes the supply chain profit, modelling the system as a single decision maker.
- The decentralized model assumes that each stakeholders wants to optimize its own profit *i.e.* each actors is a decision maker. The sum of all stakeholders profit is said to be the profit of the decentralized model.

The profit generated by the centralized and the decentralized model can then be compared.

3.1 Centralized linear program $\mathcal{C}$

The centralized model $\mathcal{C}$ aims at maximizing the whole supply chain profit *i.e.*, the sum of all actors profit, for all products (wood, lumber and paper).

Since chips are necessarily produced during the sawmilling process (MCP constraint), sawmills are allowed to throw away a part of them (*e.g.*, if there is no demand for this co-product or if the quality obtained is too poor to be used in other processes). Consequently, chips flow equations include some additional variables : $L_{i,t}$ and Lp_t, which are the chips lost for the sawmill i and the paper mill, respectively.

$\mathcal{C}$ can be defined as follows:

$$\max \overbrace{\sum_{t}^{T}\sum_{i}^{S}(Z_{i,t}\times PB_i)}^{\text{lumber sale}} + \overbrace{\sum_{t}^{T}(Y_t\times PP)}^{\text{paper sale}} - \overbrace{\sum_{t}^{T}\sum_{i}^{S}(IO_{i,t}\times HO_i + IS_{i,t}\times HS_i)}^{\text{lumber and chips storage at sawmills}}$$

$$-\underbrace{\sum_{t}^{T}\sum_{i}^{S}(W_{i,t}\times WP_i)}_{\text{wood production}} - \underbrace{\sum_{t}^{T}\left(\sum_{i}^{S}(IC_{i,t}\times HC_i) + IP_t\times HP\right)}_{\text{paper and chips storage at the paper mill}} - \underbrace{\sum_{t}^{T}\sum_{i}^{S}(XT_{i,t}\times TC)}_{\text{pulp and paper production}} \tag{1}$$

s.t.

$$Z_{i,t} \leq d_{i,t} \qquad \forall i \in S, \forall t \in T \tag{2a}$$

$$W_{i,t} \leq K_i \qquad \forall i \in S, \forall t \in T \tag{2b}$$

$$W_{i,t} = XP_{i,t} + ZZ_{i,t} \qquad \forall i \in S, \forall t \in T \tag{2c}$$

$$W_{i,t} \times MCP \leq XP_{i,t} \qquad \forall i \in S, \forall t \in T \tag{2d}$$

$$XP_{i,t} + IS_{i,t-1} + L_{i,t} = IS_{i,t} + X_{i,t} \qquad \forall i \in S, \forall t \in T \tag{2e}$$

$$ZZ_{i,t} + IO_{i,t-1} = IO_{i,t} + Z_{i,t} \qquad \forall i \in S, \forall t \in T \tag{2f}$$

$$Y_t \leq dp_t \qquad \forall t \in T \tag{3a}$$

$$\sum_{i}^{S} XT_{i,t} = YY_t \qquad \forall t \in T \tag{3b}$$

$$\sum_{i}^{S}(XT_{i,t} \times Q_i) \geq YY_t \times MCQ \qquad \forall t \in T \tag{3c}$$

$$X_{i,t} + IC_{i,t-1} + Lp_t = IC_{i,t} + XT_{i,t} \qquad \forall i \in S, \forall t \in T \tag{3d}$$

$$YY_t + IP_{t-1} = IP_t + Y_t \qquad \forall t \in T \tag{3e}$$

$$X_{i,t}, XP_{i,t}, W_{i,t}, IO_{i,t}, IS_{i,t}, Z_{i,t}, ZZ_{i,t} \in \mathbb{R} \qquad \forall i \in S, \forall t \in T \tag{4a}$$

$$X_{i,t}, IP_{i,t}, IC_{i,t}, XT_{i,t}, Y_{i,t}, YY_{i,t}, \in \mathbb{R} \qquad \forall i \in S, \forall t \in T \tag{4b}$$

The objective function is defined by (1) and aims at maximizing the sales of lumber and paper, while minimizing all production, transformation and storage costs.

Constraints from (2a) to (2f) are the sawmill constraints. The sawmills cannot sell more lumber than the market demand (2a). The sawmilling process is constrained by the sawmill capacity (2b). There is no waste nor product creation during the sawmilling process (2c). Since it is impossible to only get lumber from trees (*i.e.*, divergent process), a minimum amount of chips have to be produced (2d). Equation (2e) concerns the sawmill chip flow constraint, after sawmilling. Equation (2f) is the sawmill lumber flow constraint, after sawmilling.

Constraints from (3a) to (3e) reflect the paper mill constraints. The paper mill cannot sell more paper than the market demand (3a). One m^3 of chips is used for producing one m^3 of paper (3b). Paper quality must be at a minimum given quality (3c). Equation (3d) is the paper mill "input" chip flow constraint, before the production process. Equation (3e) is the paper mill "output" paper flow constraint, after the paper making process.

The range of values for sawmills and paper mill variables are defined by constraints (4a) and (4b), respectively.

This linear program assumes that there is a single decision maker, aiming at maximizing the total profit. The next section proposes decentralized linear programs, to optimize planning decisions of each stakeholder.

3.2 Sawmill decentralized linear program

Each sawmill tries to maximize its own profit generated from both lumber and chips sale. Therefore, from the sawmill point of view, the relevant constraint to consider are constraints (2a) to (2f), plus the variables definition constraint (4a). In order to optimize its profit, a single sawmill i has to solve the following mathematical problem:

$$\max \overbrace{\sum_{t}^{T}(Z_{i,t} \times PB_i)}^{\text{lumber sale}} + \overbrace{\sum_{t}^{T} X_{i,t} \times PC_i}^{\text{chips sale}}$$

$$- \underbrace{\sum_{t}^{T}(W_{i,t} \times WP_i)}_{\text{wood production}} - \underbrace{\sum_{t}^{T}(IO_{i,t} \times HO_i + IS_{i,t} \times HS_i)}_{\text{lumber and chips storage}} \qquad (5)$$

s.t.
(2a), (2b), (2c), (2d), (2e), (2f), (4a)

The objective (5) tries to maximize the profit *i.e.*, lumber and chips sales, minus wood production and storage costs.

Here it is considered that the paper mill buys all chips produced by the sawmills. In a following section the relevance of such a model is discussed.

3.3 paper mill decentralized linear program

In a decentralized supply chain, the paper mill focuses on improving its own profit. In this case, the relevant constraint to take into account are constraint (3a) to (3e), as well as the variables definition constraint (4b). In order to optimize its profit, the paper mill has to solve the following mathematical problem:

$$\max \quad + \overbrace{\sum_{t}^{T}(Y_t \times PP)}^{\text{paper sale}} - \overbrace{\sum_{i}^{S}\sum_{t}^{T} o_{i,t} \times PC_i}^{\text{chips purchasing}}$$

$$- \underbrace{\sum_{t}^{T}\left(\sum_{i}^{S}(IC_{i,t} \times HC_i) + IP_t \times HP\right)}_{\text{paper and chips storage}} - \underbrace{\sum_{t}^{T}\sum_{i}^{S}(XT_{i,t} \times TC)}_{\text{pulp and paper production}} \tag{6}$$

(3a), (3b), (3c), (3d), (3e), (4b)

The objective (6) aims at maximizing the profit *i.e.*, paper sales minus costs related to chips purchase and storage costs and paper production.

Here it is considered that the chips ordered by the paper mill have been produced and are available for sale. In a following section the relevance of such a model is discussed.

3.4 Discussion on profit

The profitability of the different products is first analysed, using values given in Table 2. For that purpose, all the products production profits (lumber, chips and paper) are calculated considering a given stakeholder. The considered stakeholders are the supply chain (if the model is centralized), the sawmill and the paper mill.

When looking at the whole **supply chain**, an interval for the **paper** profit is computed. The lower bound is set as the paper is fully produced from additional chips. The upper bound is computed based on the hypothesis that the paper is fully produced from existing chips.

The **paper** profit generated from additional chips can be calculated as follows :

profit = paper sale – wood processing – pulp and paper production
= $250 - 105 - 156 =$ **-11**$\$/m^3$

Sawmilling when paper is the only product is not profitable for the supply chain.

The **paper** profit generated from existing chips can be calculated as follows :

profit = paper sale – pulp paper production
= $250 - 156 =$ **94**$\$/m^3$

The MCP constraint forces to produce a minimum amount of chips. Considering that the MCQ is satisfied, producing paper from existing chips is profitable.

We can therefore estimate that profitability for producing paper from one m^3 is between 94\$ and -11\$.

Remark : consider that MCQ is satisfied. To be profitable for the supply chain, the paper must be made with a given minimum proportion of existing chips (*i.e.* produced by the MCP constraint). This value can be calculated. Consider x

the minimum proportion of existing chips.

profit $= \text{existing chips cost} \times x + \text{additional chips cost} \times (1 - x) \leq \text{paper profit}$
$= 0 \times x + 105 \times (1 - x) \leq 94 \Leftrightarrow x \geq \frac{11}{105}$

The existing chips proportion is $\frac{11}{105} = 10.5\%$. This means that for each $0.105m^3$ of existing chips, it is profitable to produce an additional $0.895m^3$ of chips to make $1m^3$ of paper.

The products profitability for the **sawmills** can be estimated as follows :
Profit for **chips** :

profit $= \text{chips sale} - \text{wood processing}$
$= 56 - 105 =$ **-49**$\$/m^3$

Wood only used for chips production is not profitable for the sawmills. In a decentralized model, without any incentive, a sawmill has no interest to produce more chips than the MCP constraint.

Profit for **lumber**, considering the MCP constraint

profit $= \text{lumber sale} \times (1 - MCP) - \text{wood processing}$
$= 142 \times 90\% - 105 =$ **22.8**$\$/m^3$

Focusing on sawmilling to produce the maximum amount of lumber is profitable, even without the chips sales.

If we then look at the **paper mill**, the profit for **paper** can be estimated as follows:

profit $= \text{paper sale} - \text{chips purchase} - \text{paper production}$
$= 250 - 56 - 156 =$ **38**$\$/m^3$

Considering that the MCQ is satisfied, chips are profitable for the paper mill. In a decentralized model, paper mill has interest to produce and sell as much as paper as possible, while the MCQ holds.

Since in this study, cost are known and constant, the above properties hold in the whole paper.

4 Centralized model analysis

This section investigates an unique decision maker using the centralized model. The special case considering constant demand is also studied.

4.1 Constant demands

All the demands are now supposed to be constant. Taking this assumption allows to understand the properties that hold in the centralized case. In this context, there is no interest to store any product. In fact, the problem $\mathcal{C}$ can be seen as a succession of identically and separated mono-period problems. Since costs are known and constant, the properties of the Section 3.4 still hold. The key decision for the supply chain is to know what optimal quantity of chips should be used for paper, for each quality. To calculate that, a mathematical program can be written, focusing on chips variables. It is a mono-period mathematical program called $\mathcal{P}_{\mathrm{L}}$.

4.1.1 Mathematical program $\mathcal{P}_{\mathbf{L}}$

$\mathcal{P}_{\mathrm{L}}$ is found out by simplification of $\mathcal{C}$. As said previously, there is no interest to store any product, which leads to an elimination of all the inventory variables. Since the resulting mathematical program is mono-period, the flow equations can be simplified in variables equivalences. For instance, the flow constraint 3d which is :

$$X_{i,t} + IC_{i,t-1} \geq IC_{i,t} + XT_{i,t} \qquad \forall i \in S, \forall t \in T$$

It can be turned into :

$$X_{i,t} \geq XT_{i,t} \qquad \forall i \in S$$

In fact, it is a mono-period problem, and therefore the paper mill cannot store chips, it has no interest to buy additional quantity, which are destined to be thrown away. As a result :

$$X_{i,t} = XT_{i,t}$$

All these simplifications lead to keep the following variables, $XP_{i,t}$ and $X_{i,t}$. These variables represent the quantity of chips a sawmill should produce, and the quantity of chips that should be delivered to the paper mill.

After these variable eliminations, the focus is on the objective function. Discarding all the storage costs, considering mono-period problem, the objective function of $\mathcal{C}$ becomes :

$$\max \overbrace{\sum_{i}^{S}(Z_{i,t} \times PB_i)}^{\text{lumber sale}} + \overbrace{Y_t \times PP}^{\text{paper sale}} - \overbrace{\sum_{i}^{S}(W_{i,t} \times WP_i)}^{\text{wood production}} - \overbrace{\sum_{i}^{S}(XT_{i,t} \times TC)}^{\text{pulp paper production}}$$

in which $Y_{i,t}$ and $XT_{i,t}$ can be turned into $X_{i,t}$ (for the reason explained above). Furthermore $W_{i,t}$ can be replaced by $XP_{i,t}$, because WP_i is the cost for producing any additional chips (beyond the MCP). Also, the term $\sum_i^S (Z_{i,t} \times PB_i)$ can be discarded because lumber demand is not linked to chips flows. This leads to another objective function, which is :

$$\max \quad \overbrace{\sum_{i}^{S} X_{i,t}(PP - TC)}^{\text{profit with existing chips}} \quad - \quad \overbrace{\sum_{i}^{S} XP_{i,t} \times WP_i}^{\text{cost to produce additional chips}}$$

The constraints to consider are therefore the MCP, the MCQ, the sawmill capacity, and the paper demand.

For a single period t, the mono-period mathematical program $\mathcal{P}_{\mathrm{L}}$ is defined below:

$$\max \quad \sum_{i}^{S} X_{i,t}(PP - TC) - \sum_{i}^{S} XP_{i,t} \times WP_i \tag{7}$$

s.t.

$$XP_{i,t} \geq \min\left\{\frac{\mathrm{MCP}}{1-\mathrm{MCP}} \times d_{i,t}, \mathrm{MCP} \times K_i\right\} \qquad \forall i \in S \tag{8a}$$

$$\sum_{i}^{S}(X_{i,t} \times Q_i) \geq \sum_{i}^{S}(X_{i,t} \times \mathrm{MCQ}) \tag{8b}$$

$$XP_{i,t} \leq K_i \qquad \forall i \in S \tag{8c}$$

$$\sum_{i}^{S} X_{i,t} \leq dp_t \tag{8d}$$

$$X_{i,t} \leq XP_{i,t} \qquad \forall i \in S \tag{8e}$$

$$X_{i,t}, XP_{i,t} \in \mathbb{R}^+ \qquad \forall i \in S \tag{8f}$$

The amount of lumber produced by a sawmill is bounded either by its capacity, or by the lumber demand. Considering the MCQ, the minimum amount of chips produced by a sawmill is a fraction of either its capacity or the lumber demand. This minimum amount is given by the constraint (8a). Even if this constraint is not linear, it can be transformed into a linear constraint, adding an additional variable. Thus this mathematical program can be written as a linear program.

Constraint (8b) (MCQ) verifies that the quality of the chips used is at least the minimum paper quality. The chips produced are also bounded by the capacity of the sawmills (Constraint (8c)). The constraint (8d) bounds the amount of chips used to the paper demand. Finally, constraints (8e) and (8f) ensure that the variables are well defined and linked.

The mathematical program $\mathcal{P}_{\mathrm{L}}$ gives the optimal amount of chips produced and used for a single period. Even in a multi-period problem, in the case where demands are constant, the optimal amount of chips produced and used can be provided either by $\mathcal{P}_{\mathrm{L}}$ or $\mathcal{C}$.

5 Decentralized model with contracts

In this section, different decentralized scenarios are investigated. Demands are not constant anymore and may vary. All the problems considered are therefore multi-periods. However, the mono-period results above are used to design contracts at the end of this section.

This decentralized problem $\mathcal{D}$ aims at maximizing each stakeholder's own profit. In this purpose, at each period t, each stakeholder successively optimizes its own planning on a rolling horizon of H periods, *i.e.* from t to $t + H - 1$.

5.1 Imbalance of extreme decentralized cases

The first two scenarios show that in a decentralized case, the lack of regulation or contract leads to a profit loss for the supply chain. A new variable $o_{i,t}$ is introduced to reflect the amount of chips ordered by the paper mill at the sawmill i, at period t.

The two scenarios are :

- A decentralized supply chain where the paper mill is dominant, *i.e.* the paper mill chooses the quantity of chips to buy from the sawmills (Algorithm 1).
- A decentralized supply chain where the sawmills are dominant, *i.e.* the sawmills choose the quantity of chips to produce, and then the paper mill chooses to buy an amount lesser or equal than the available amount of chips (Algorithm 2).

Algorithm 1 A decentralized algorithm with dominant paper mill

```
for each period $t$ : do
    The paper mill optimizes its planning on the rolling horizon $H$.
    Then it sends $H$ orders $o_{i,t}$ for the next $H$ periods for each sawmill $i$, $\sum_{t'=1}^{t} o_{i,t'} \leq t \times K_i$.
    for each sawmill $i$ : do
        The sawmill $i$ optimizes its own planning regarding the amount of chips $X_i = o_{i,t}$ to provide to the paper mill.
    end for
end for
```

Let the paper mill be the dominant (Algorithm 1). At each period, the paper mill orders a certain amount of chips $o_{i,t}$ that the sawmill i has to satisfy (the capacity of the sawmill is respected, *i.e.* $\sum_{t'=1}^{t} o_{i,t'} \leq t \times K_i$). A unique chips production is not profitable for the sawmills, so any chips quantity ordered beyond the MCP constraint can be a profit waste. The paper is profitable for the paper mill, so depending on the paper demand, a large quantity of chips could be ordered. Moreover, since the paper profit for the whole supply chain is also negative (see Section 3.4), the supply chain profit overall decreases.

If the sawmills are dominant (Algorithm 2), they produced a given amount of chips. Afterwards, the paper mill chooses what quantity to buy. As said previously, sawmills must respect the MCP constraint, forcing them to produce a minimum quantity of chips. Also, any quantity of chips produced beyond the MCP is a profit waste for a sawmill. Thus the sawmills will only produced the minimum quantity of chips mandatory. Indeed there is no incentive for the sawmills to produce more than the MCP.

Algorithm 2 A decentralized algorithm with dominant sawmills

```
for each period $t$ : do
  for each sawmill $i$ : do
    On a rolling horizon of $H$ periods, the sawmill $i$ optimizes its planning, producing an amount of chips $XP_{i,t}$. The paper mill buys a quantity of chips $X_{i,t}$ to the paper mill, $X_{i,t} \leq XP_{i,t}$.
  end for
  On a rolling horizon of $H$ periods, the paper mill optimizes its planning based on the $X_{i,t}$ provided by the sawmills.
end for
```

Remind that the MCQ has to be respected when paper is produced, involving that certain quantity of chips may be thrown away. However, it would be profitable for the whole supply chain to produce additional high quality chips to produce more paper, using existing chips (see Section 3.4). This situation does not allow additional chips production, leading to a non-optimal global solution.

To conclude, both these extreme decentralized situations are not convenient. In the next section, a decentralized scenario with contract is investigated.

5.2 Contract design

The next scenario proposed a more balanced and flexible decentralized supply chain dynamic. The previous mathematical program $\mathcal{P}_{\mathrm{L}}$ defines the optimal quantity of chips to order, but only in the case where demands are known and constant. Considering a multi-period problem with varying demands, contracts have to be fixed and constant on the whole horizon in order to guide the different stakeholders in the decisions they make.

Hellion *et al.* [17] developed the stability contracts for securing a retailer's supply while ensuring a beneficial relationship for the stakeholders (retailer and suppliers). These stability contracts initially constrained the retailer order in two ways :

- by defining minimum and maximum bounds on orders amounts (denoted L and U, respectively);
- by defining dynamic time windows for each orders.

Based on Hellion *et al.* [17] work, stability contracts are used for better satisfying paper mill needs. However, since each period corresponds to a possible delivery, the time discretization does not allow to define any dynamic time windows.

Consequently, the stability contract must define L_i and U_i for each sawmill i. Then, for each period t, $L_i \leq o_{i,t} \leq U_i$. L_i and U_i values are computed according to the sawmills capacity and lumber demand. Sawmills must also provide a quantity of chips $X_{i,t}$ such as $X_{i,t} = o_{i,t}$. Algorithm 3 presents how the supply chain works in the multi-period decentralized context.

Algorithm 3 The decentralized procedure $\mathcal{D}$

```
for each sawmill $i$ : do
  The paper mill and the sawmill $i$ agree on both $L_i$ and $U_i$ bounds.
end for
for each period $t$ : do
  The paper mill optimizes its planning on the rolling horizon $H$, calculating the values $o_{i,t'}$, $t' \in \{t \dots t + H - 1\}$ , such as $L_i \leq o_{i,t'} \leq U_i$.
  The paper mill sends $H$ orders $o_{i,t'}$ at each sawmill $i$.
  for each sawmill $i$ : do
    The sawmill $i$ optimizes its own planning taking into account the paper mill's orders.
    The sawmill $i$ provides an amount of chips $X_{i,t}$ to the paper mill, such as $X_{i,t} = o_{i,t}$, for every $H$ next periods.
  end for
end for
```

Remark : Since demands are not constant in this section, the average demand is used to calculate the contract parameters. The average demand of paper is noted dp. Furthermore, the average demand for lumber at each sawmill i is noted d_i.

The output values of $\mathcal{P}_\mathrm{L}$ are the optimal quantities of chips to order so as to satisfy the demand for a single period, a multi-period problem when demands are constant. However, in a decentralized multi-period problem with varying demands, this can be seen as a lower bound of chips for the paper mill at each period, without decreasing the profit of the supply chain. Then the output values X_i of $\mathcal{P}_\mathrm{L}$ can be assigned to L_i.

Definition : $\min_i$ is the minimum of chips produced by the sawmill i, according to its lumber market and its capacity.

$$\min_i = \min \left\{ \frac{\mathrm{MCP}}{1 - \mathrm{MCP}} \times d_{i,t}, \mathrm{MCP} \times K_i \right\}$$

Remind that if low quality chips are available, it is profitable for the supply chain to produce additional high quality chips to make paper (see Section 3.4). Thus the upper bound U_i should include all chips produced by the sawmill i, according to the MCP constraint. The likely insufficient quality of this mix of chips must be improved by producing additional high quality chips, in order to reach the required quality (MCQ). Formally, U_i defines for each sawmill i the minimum quantity of chips satisfying :

- $U_i \geq \min_i \; \forall i \in S$;
- $\sum_i^S U_i Q_i \geq \sum_i^S U_i \mathrm{MCQ}$.

This can be calculated by a similar mathematical program than $\mathcal{P}_{\mathrm{L}}$. Since the demands are no longer constant, it would be profitable to order more chips to later satisfy a large paper demand. Furthermore, U_i values can be computed using $\mathcal{P}_{\mathrm{L}}$, but the constraint (8d) ensuring that the amount of chips provided does not exceed the paper demand for one period, must be discarded. This new mathematical program is called $\mathcal{P}_{\mathrm{U}}$ and is defined as follows :

$$\max \; \sum_i^S X_{i,t}(PP - TC) - \sum_i^S XP_{i,t} \times WP_i$$

s.t. (8a) (8b) (8c) (8e) (8f)

The values for U_i are computed based on the output values X_i of this mathematical program.

Remark : Since the only difference between $\mathcal{P}_{\mathrm{U}}$ and $\mathcal{P}_{\mathrm{L}}$ is the constraint (8d), if dp is large, the output from $\mathcal{P}_{\mathrm{U}}$ and $\mathcal{P}_{\mathrm{L}}$ are the same. Formally, if $dp \geq \sum_{i \in S} U_i^*$, the constraint (8d) has no impact on the formulation and consequently $L_i^* = U_i^*$, $\forall i \in S$.

To summarize, for each sawmill i, the values for L_i and U_i can be computed using $\mathcal{P}_{\mathrm{L}}$ and $\mathcal{P}_{\mathrm{U}}$, respectively. Moreover, the solutions of the mathematical programs $\mathcal{P}_{\mathrm{L}}$ and $\mathcal{P}_{\mathrm{U}}$ present some properties :

Property 1. *Optimal solution of $\mathcal{P}_U$ determines the value U_i to its min_i, except for a set of chips S (which only include the chips with the best quality).*
The solution follows the form below. Sort all the sawmills i by Q_i in descending order :

- $\forall i \in \{1; \ldots ; k-1\}$, $U_i = K_i$
- $U_k = \frac{\sum_{i=1}^{k-1} U_i(MCQ - Q_i)}{Q_k - MCQ}$
- $\forall i \in \{k+1; \ldots ; S\}$, $U_i = min_i$

Remark : *the proof explains how to find k.*

Proof. The proof is given in the appendix. □

The property 1 and its proof leads to a simple procedure $\mathcal{R}_{\mathrm{U}}$ (Algorithm 4) which computes the optimal solution of $\mathcal{P}_{\mathrm{U}}$. For each sawmill i the value of U_i can be assigned via the output of the procedure $\mathcal{R}_{\mathrm{U}}$.

Algorithm 4 procedure $\mathcal{R}_{\mathrm{U}}$ computing the optimal solution of $\mathcal{P}_{\mathrm{U}}$

```
for each $i$ from 1 to $S$ do
  calculate $L_k$ as if $i = k$
  $k$ is the first that satisfies $L_k \leq K_k$
end for
calculate $L_k^*$.
```

$\mathcal{P}_{\mathrm{L}}$ presents some properties as well. The MCQ is a constraint in the formulation, and since all the chips have the same price, an infinity of optimal solutions exists. However these solutions do not have the same final quality of chips, because MCQ is a constraint, not an objective. The purpose is to find, among the optimal solutions, the solution with the best quality of paper.

The following property (property 2) defines the structure of an optimal solution for $\mathcal{P}_{\mathrm{L}}$, the one with the best quality of paper.

Property 2. *Sort all sawmills i by Q_i in descending order. The optimal solution of $\mathcal{P}_L$ which maximizes the average paper quality is the following form:*

- $\forall i \in \{1; \ldots ; k-1\}$, $L_i = K_i$
- $L_k = \frac{dp(MCQ-Q_m)+(Q_m-Q_i)(\sum_{i=1}^{k-1} K_i+\sum_{i=k+1}^{m-1} min_i)}{Q_k-Q_m}$
- $\forall i \in \{k+1; \ldots ; m-1\}$, $L_i = min_i$
- $L_m = \{dp - \sum_{j \neq m}^{j \in S} L_j\}$
- $\forall i \in \{m+1; \ldots ; S\}$, $L_i = 0$

Remark : *the proof explains how to find k and m.*

Proof. The proof is given in the appendix. □

Property 2 and its proof leads to a simple procedure $\mathcal{R}_{\mathrm{L}}$ (Algorithm 5) which computes the optimal solution of $\mathcal{P}_{\mathrm{L}}$. For each sawmill i, the value of L_i can be assigned to the output of the procedure $\mathcal{R}_{\mathrm{L}}$.

For each sawmill i, the contract is created by assigning at L_i and U_i the outputs of $\mathcal{R}_{\mathrm{L}}$ and $\mathcal{R}_{\mathrm{U}}$, respectively.

5.3 Experiments

In this section, decentralized procedure $\mathcal{D}$ with stability contracts is compared to the centralized procedure $\mathcal{C}$, using instances with demands following a normal distribution. For each sawmill i, stability contracts are based on parameters L_i and U_i computed by procedures $\mathcal{R}_{\mathrm{L}}$ and $\mathcal{R}_{\mathrm{U}}$

Algorithm 5 procedure $\mathcal{R}_{\mathrm{L}}$ computing the solution with best paper quality among the optimal solutions of $\mathcal{P}_{\mathrm{L}}$

```
for each $i$ from 1 to $S$ do
  calculate $L_k$ as if $i = k$
  $k$ is the first that satisfy $L_k \leq K_k$
end for
for each $i$ from $k + 1$ to $S$ do
  with the value of $L_k$, calculate the final paper quality
end for
keep the $m$ that maximize the paper quality.
calculate $L_k^*$.
calculate $L_m^*$.
```

Different groups of instances are generated, and they differ by lumber and paper average demands. The capacities of the sawmills, which are K, $0.5K$ and K, are fixed to 90, 45 and 90, respectively. The lumber demand can be much larger than the paper demand, and inversely. Also, the capacity of the sawmills can be significant or not. That leads to 4 groups, as shown in Table 5, that encompass the average demand for each stakeholder or product. In each group, 20 different instances are generated.

Table 5: Instance parameters

Group	$d_{1,t}$	$d_{2,t}$	$d_{3,t}$	dp_t
G1	60	30	60	10
G2	30	15	30	50
G3	120	60	120	20
G4	120	60	120	150

These instances are tested in two experiments. First, the profit of each stakeholders is evaluated using a given fixed variance (20%) (Section 5.3.1). The global profit is then investigated for a variance varying, from 5% to 50%, with a step of 5 (section 5.3.2).

5.3.1 Comparison of each stakeholder's profit using fixed variance

For each group, the variance is fixed at 20% of the average demand. Results are displayed in Table 6.

Results shows that the profit obtained with the proposed method are close to the ones generated using the centralized method (lower than 1%). Certain values are even better in the proposed method than with the centralized approach, which can be explained by a different profit distribution. However, all values are close to their optimum. The overall profit of the decentralized supply chain is 99.3% the

Table 6: Supply chain member's profit with fixed variance

Group	Scenario	Average profit for the whole group					Chips proportion		
		P	S_1	S_2	S_3	Total	S_1	S_2	S_3
G1	$\mathcal{C}$	46 450	76 287	38 078	75 864	236 679	0.10	0.10	0.10
	$\mathcal{D}$	46 436	76 170	37 774	**73 599**	**233 980**	0.10	0.10	**0.11**
G2	$\mathcal{C}$	47 956	38 195	19 041	27 577	132 769	0.10	0.10	0.16
	$\mathcal{D}$	47 953	37 775	18 815	27 586	132 129	0.10	0.10	0.16
G3	$\mathcal{C}$	92 225	102 912	51 513	97 277	343 926	0.10	0.10	0.11
	$\mathcal{D}$	92 244	102 856	51 516	**93 329**	**339 945**	0.10	0.10	**0.12**
G4	$\mathcal{C}$	129 462	103 037	51 504	65 380	349 383	0.10	0.10	0.16
	$\mathcal{D}$	129 473	103 038	51 497	65 319	349 327	0.10	0.10	0.16

centralized profit. In Table 6, poor local results (more than 1% lower than the centralized profit) are displayed in bold. Those results correspond to the cases where the paper demand is low for sawmills having the best chips quality. Decentralized decision making for groups G2 and G4 is very close to the optimal. Considering that these two groups face a large paper demand, contracts seems to be particularly effective in that case.

The last three columns of the table display the proportion of chips produced for each sawmill. When the value is greater than 0.1, additional chips have to be produced. The chips proportion of sawmill S_3 is 0.16 for G2 and G4, which are the two groups facing a large demand for paper.

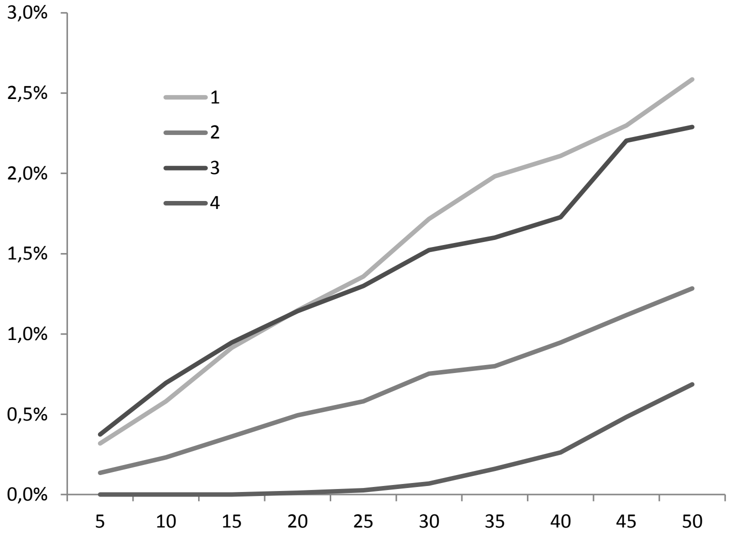
5.3.2 Comparison of the whole supply chain profit when the variance is increased

The next experiment aims at comparing the profit of the whole supply chain generated from the centralized and decentralized model using an increasing variance. In particular, the variance starts at 5, up to 50, and increases by a step of 5. For each variance and each group, 100 instances are solved using $\mathcal{C}$ and $\mathcal{D}$. In total, 800 instances are solved.

For each group of instances, the profit difference increases with the variance. Concerning the decentralized model, contracts are generated based on the average demand for paper and lumber. However, the average demand are less and less indicative as variance increases. For a variance of 50%, the profit difference almost reach 3% for the worst groups G1 and G3. On the other hand, considering the group G2 and G4, even with the largest variance, the profit difference between the centralized and the decentralized system is at 1% and 0.5%, respectively.

In summary, the experiments based in stakeholder's profit show that the supply chain profit is correctly distributed among the stakeholders because their profits

Figure 3: Comparison of the whole supply chain profit, for each group of instances



are close between the centralized and the decentralized model. The experiment using an increasing variance demonstrates that stability contracts can optimize the supply chain profit, even with large variances on the demands. However, to be effective, these contracts need a good assessment of the current average demand of the market. Furthermore, each change in the average demand should involve a modification of the contract terms to ensure fair distribution of the supply chain profit. By using stability contracts, forest product companies of the Côte-Nord region could therefore both better respond to the paper mill demand while improving coordination between supply chain operations.

6 Conclusion

In this paper, we investigate the case of a paper mill and its three suppliers, which are sawmills. All these companies are located in the Côte-Nord region, in Quebec, Canada. The purpose is on securing the paper mill supplies by creating beneficial contracts for all stakeholders. The paper mill and the sawmills are modeled, with their costs, capacities, and chips quality. Two industrial constraints are considered, reflecting the divergent production process and the quality requirements for paper production, leading to a specific problem. These constraints are then used to design particular contracts between the paper mill and each sawmill.

Two contexts are presented and compared : the centralized environment and the decentralized decision making process. Lumber and chips market are considered in the models to take into account industry's reality. Algorithm used to create the specific contracts are provided, leading to a practical solution. An experimental study shows that the profit of the decentralized model, managed by the contracts, is 99.3% the centralized profit. Furthermore, each individual profit under contract is close to the centralized optimal solution. Another experiment shows the efficiency of the stability contracts, even for largest demand variances.

The experiments showed that if a good assessment of the average demand (for both paper and lumber market) is conducted, the stability contracts could be effective for the whole supply chain, as well as for each stakeholder. The key of the problem is therefore to get a good assessment of the future average demand while determining and negotiating the contracts terms efficiently. In that purpose, the Côte-Nord stakeholders should share the necessary informations to get the best possible demand forecast.

This case study has served well to propose a contract design methodology. Generalization of the methodology to divergent process industries such as those found in refinery or agricultural industry could be done.

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7 Appendix

Proof. of property 1.

This proof is presented in three parts. First, we demonstrate that there always exists a solution that contains only one index k, such as U_k^* is a fractional value, *i.e.* $U_k^* > \min_k$ and $U_k^* < K_k$. Then, we show how to calculate U_k^*. Finally (with the sawmills sorted by Q_i in descending order), we prove that this formula applied on each i such as $i < k$ leads to $U_i > K_i$.

Preliminary remark : $Q_k >$ MCQ, because there is no reason to produce chips below the MCQ.

Say that it exists an optimal solution such as there is two fractional value, say U_a^* and U_b^*, with $a < b$. In this case we can reallocate the value $\alpha = U_b^* - \min_b$ from b to a. If $U_a^* + \alpha \geq K_a$, assign at U_a^* the value K_a, reassign the rest in U_b^* and thus there is only one fractional value, which is U_b^*. Otherwise, if $U_a^* + \alpha < K_a$, there is only one fractional value, which is U_a^*, and the chips quality of the new solution is better.

The value U_k^* is found below :

$$\sum_{i=1}^{S} U_i Q_i = \sum_{i=1}^{S} U_i \text{MCQ}$$

$$\sum_{\substack{i \neq k}}^{i \in S} U_i Q_i + U_k Q_k = \sum_{\substack{i \neq k}}^{i \in S} U_i \text{MCQ} + U_k \text{MCQ}$$

$$U_k = \frac{\sum_{i \neq k}^{i \in S} U_i(\mathrm{MCQ} - Q_i)}{Q_k - MCQ}$$

If this formula is used on a given i, such as $i < k$, this means that $U_k = \min_k$, and $U_i < K_i$. In this case, there is a chips quality loss and the only way to satisfy the MCQ is to have a $U_i > K_i$, which is a contradiction. Thus, all i have to be tried, from 1 to S, and the first i satisfying $U_i \leq K_i$ means $i = k$. $\square$

Proof. of property 2.

This proof is presented in three parts. First, we show that the optimal solution of $\mathcal{P}_{\mathrm{L}}$ which maximizes the average paper quality admits at most one m and one k, such as $0 < L_m^* < \min_m$ and $\min_k < L_k^* < K_k$. Then, we demonstrate how to compute L_m^* and L_k^*. Finally (with the sawmills sorted by Q_i in descending order), we show how to find m and k.

Say that it exists an optimal solution which maximizes the average paper quality such as there are two indexes, say k and k', with $k < k'$. These indexes both satisfy $\min_k < L_k^* < K_k$ and $\min_{k'} < L_{k'}^* < K_{k'}$. By reallocating a small value from k' to k, the average paper quality increases, which is a contradiction. Similarly, there are two other indexes, say m and m', with $m < m'$, $0 < L_m^* < \min_m$ and $0 < L_{m'}^* < \min_{m'}$. By reallocating a small value from m' to m, the average paper quality increases, which is a contradiction.

The purpose of the value of L_m^* is to meet the paper demand dp. In the case where m exists (see the third part of the proof):

$$L_m^* = dp - \sum_{j \neq m}^{j \in S} L_j$$

The purpose of the value of U_k^* is to maximize the paper quality :

$$\sum_i^S L_i Q_i = \sum_i^S L_i \mathrm{MCQ} \tag{9}$$

Note that $\forall i \in [1; \ldots ; k-1]$, $L_i = K_i$ and $\forall i \in [k+1; \ldots ; m-1]$, $L_i = \min_i$ and $L_m = dp - \sum_{i \neq m}^S$ and $\forall i \in [m+1; \ldots ; S]$, $L_i = 0$.

$$(9) \Leftrightarrow \sum_{i=1}^{k-1} K_i Q_i + L_k Q_k + \sum_{i=k+1}^{m-1} \min{}_i Q_i + Q_m(dp - \sum_{i=1}^{k-1} K_i - L_k - \sum_{i=k+1}^{m-1} \min{}_i)$$

$$= \sum_{i=1}^{k-1} K_i \mathrm{MCQ} + L_k \mathrm{MCQ} + \sum_{i=k+1}^{m-1} \min{}_i \mathrm{MCQ} + \mathrm{MCQ}(dp - \sum_{i=1}^{k-1} K_i - L_k - \sum_{i=k+1}^{m-1} \min{}_i)$$

After development and simplifications:

$$(9) \Leftrightarrow L_kQ_k - L_kQ_m$$

$$= -\sum_{i=1}^{k-1} K_iQ_i - \sum_{i=k+1}^{m-1} \min{}_iQ_i - dpQ_m + \sum_{i=1}^{k-1} K_iQ_m + \sum_{i=k+1}^{m-1} \min{}_iQ_m + dp\mathrm{MCQ}$$

After rearrangement and factoring:

$$(9) \Leftrightarrow L_k = \frac{dp(\mathrm{MCQ} - Q_m) + (Q_m - Q_i)(\sum_{i=1}^{k-1} K_i + \sum_{i=k+1}^{m-1} \min_i)}{Q_k - Q_m}$$

The third part of the proof follows :

If this formula is used on a given i, such as $i < k$, this means that $U_k = \min_k$, and $L_i < K_i$. In this case, there is a chips quality loss and the only way to satisfy the MCQ is to have a $L_i > K_i$, which is a contradiction. Thus, all i have to be tried, from 1 to S, and the first i satisfying $L_i \leq K_i$ means $i = k$.

The value for L_k is dependent of the m chosen. The formula for L_k guarantees that considering a given m, L_k maximizes the quality of paper. Thus for each possible m (between $k+1$ and S) L_k has to be calculated. The final m is the one that maximizes the quality, *i.e.* $\sum_{i=1}^{S} L_iQ_i$. □

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